Lebanese American University **Spring 2006**

Byblos

Probability and Statistics **Date:** 31/03/2006

Test #1 **Duration:** 1h 20

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1. Let X be a continuous random variable such that its density function is

$$f(x) = \begin{cases} k(x^2+1), & 0 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Evaluate k.

   (b) Find $F(x)$ and use it to evaluate $P(0.2 < X < 0.5)$.

2. We consider a sack of fruit which contains 3 oranges, 4 apples and 5 bananas. We select randomly 2 pieces of fruit. Find the probability that we obtain 1 oranges and 1 apples if:

   (a) the 2 pieces are selected at the same time.

   (b) the 2 pieces are selected in succession with replacement.

   (c) the 2 pieces are selected in succession without replacement.

   Conclusion?

3. (a) How many three-digit numbers can be formed from the digits $0, 1, 2, 3, 4$ and 5 if each digit can be used only once?

   (b) How many of these are odd numbers?

   (c) How many of these are even numbers?

   (d) How many of these are greater than 450?

   (e) How many of these are divisible by 5?

   (f) *Bonus question.* How many of these are divisible by 3 and 5?

4. We consider tow bags such that the first bag contains 5 white balls and 6 black balls and the second bag contains 8 white balls and 7 black balls. We draw one ball from the first bag and we placed unseen in the second bag. After, we draw one ball from the second bag and we placed unseen in the first bag. Finally, we draw a ball from the first bag. Find the probability that the three drawn balls have the same color.

MARKS : 1. [25] **2.** [25] **3.** [30] **4.** [20] Bonus question [10]

Lebanese American University **Summer I**

Byblos **2006**

Probability and Statistics **Date:** 14/07/2006

Test #1 **Duration:** 1h 30

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1. (a) How many three-digit numbers can be formed from the digits $0, 1, 2, 3, 4, 5$ and 6 if each digit can be used only once?
   (b) How many of these are odd numbers?
   (c) How many of these are even numbers?
   (d) How many of these are divisible by 5?
   (e) How many of these contain the digit 3?
   (f) How many of these contain the digits 5 and 6 at the same time?
   (g) How many of these contain the digits 0 and 4 at the same time?

2. We consider a bag which contains 4 red balls, 3 blue balls and 5 green balls.
   (a) We select at the same time 4 balls from this bag. Find the probability that we obtain a number of red balls equal to the number of blue balls.
   (b) *Bonus question.* Repeat question (a) in the case in which the 4 balls are selected in succession and without replacement.

3. A die is loaded in such a way that an even number is thrice as likely to occur as an odd number. We consider the following two (consecutive) statistical experiments:
   - **First experiment:** we toss the loaded die two times.
   - **Second experiment:** we flip a coin n times, where n is sum of the two numbers obtained in the first experiment.

   (a) Find the probability that we obtain a sum of 10 in the first experiment.
   (b) Find the probability that we obtain a sum of 10 in the first experiment and a number of heads equal to 9 in the second experiment.
   (c) Find the probability that we obtain a number of heads equal to 9 in the second experiment.

MARKS : 1. [40] **2.** [25] **3.** [35] Bonus question [10]

Lebanese American University **Summer II**

Byblos **2006**

Probability and Statistics **Date:** 28/09/2006
Test #1 **Duration:** 1h 30

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1. (a) How many four-digit numbers can be formed from the digits $0, 1, 2, 3, 4, 5, 6$ and 7 if each digit can be used only once?
   (b) How many of these are even numbers?
   (c) How many of these are odd numbers?
   (d) How many of these are divisible by 5?
   (e) How many of these contain the digit 2?
   (f) How many of these contain the digits 3 and 5 at the same time?
   (g) How many of these contain two odd digits and two even digits at the same time?

2. We consider three dice: one red, one blue and one green. We assume that
   - the red die is fair,
   - the blue die is loaded in such a way that an even number is twice as likely to occur as an odd number,
   - the green die is loaded in such a way that an odd number is twice as likely to occur as an even number.

   We toss the three dice. Find the probability that
   (a) we obtain the same number on the three dice?
   (b) we obtain two even numbers and one odd number on the three dice?
   (c) we obtain the number 3 exactly two times on the three dice?
   (d) *Bonus question.* we do not obtain the number 3?

3. We consider a bag which contains 5 red balls and 7 blue balls. We draw (without replacement) two balls from the bag. If the drawn balls have the same color then we put a new red ball in the bag, and if the drawn balls have different color then we put a new blue ball in the bag. Finally we draw one ball from the bag. Find the probability that the last drawn ball is red?

MARKS : 1. [40] **2.** [30] **3.** [30] Bonus question [10]

Lebanese American University **Spring 2007**

Byblos

Probability and Statistics **Date:** 05/04/2007

Test #1 **Duration:** 1h 30

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1. A fair die is tossed five times.
   (a) Find the probability that we obtain the same number in the five tosses.
   (b) Find the probability that we obtain five even numbers in the five tosses.
   (c) Find the probability that we obtain two even numbers and three odd numbers in the five tosses.

2. From the set $\{1, 2, 3, 4, ..., 144, 145\}$, we select in succession and without replacement three numbers. Find the probability that
   (a) the product of the three selected numbers is even.
   (b) the sum of the three selected numbers is even.

3. We consider a bag which contains 7 red balls and 9 blue balls. We draw (without replacement) one ball from the bag. If the drawn ball is red than we place two new blue balls in the bag, and if the drawn ball is blue than we place two new red balls in the bag. Finally we draw one ball from the bag.
   (a) Find the probability that the two drawn balls have the same color?
   (b) Given that the second drawn ball is red, find the probability that the first one is blue?

4. A die is loaded in such a way that an even number is thrice as likely to occur as an odd number. We consider the following experiment:

   "First we toss the loaded die. If we obtain an even number then we flip a coin three times, otherwise we flip the coin four times".

   (a) Find the probability that we obtain exactly two heads.
   (b) Find the probability that we obtain a number of heads equals to the number of tails.
   (c) *Bonus question.* Find the probability that the number obtained on the die equals to the number of obtained heads.

MARKS : 1. [20] **2.** [25] **3.** [30] **4.** [25] Bonus question [10]

Lebanese American University **Summer II**

Byblos **2007**

Probability and Statistics **Date:** 21/08/2007
Test #1 **Duration:** 1h 30

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1. A die is loaded in such a way that an even number is twice as likely to occur as an odd number. We toss this die five times.
   (a) Find the probability that we obtain five odd numbers in the five tosses.
   (b) Find the probability that we obtain the same number in the five tosses.
   (c) Find the probability that we obtain two even numbers and three odd numbers in the five tosses.

2. (a) How many four-digit numbers can be formed from the digits $0, 2, 3, 4, 5, 6, 7, 8$ and 9 if each digit can be used only once?
   (b) How many of these contain the digit 3?
   (c) How many of these contain two even digits and two odd digits?
   (d) How many of these contain two prime digits?
   (e) How many of these contain two prime digits and two even digits?

3. A coin is loaded such that $P(\text{Tail}) = 2P(\text{Head})$. We consider the following experiment which consists of two (consecutive) parts:
   - First we flip the loaded coin several times to obtain head for the first time.
   - Second we toss a die n times where n is the number of times that we flipped the coin in the first part.

   (a) Find the probability the we tossed the loaded coin six times in the first part of the experiment.
   (b) Find the probability the we tossed the loaded coin six times in the first part of the experiment and that the numbers obtained on the die in the second part of the experiment are all the same.
   (c) Find the probability that the numbers obtained on the die in the second part of the experiment are all the same.
   (*Hint*: We recall that for $0 < a < 1$, we have $\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}$).

4. We consider two tennis players, player1 and player2. We assume that the probability that player1 wins a point over player2 is p. We recall that:

   A ***game*** *consists of a sequence of points played with the same player serving, and is won by the first player to have won at least four points and at least two points more than his opponent. When at least three points have been scored by each side* *and the players have the same number of points, the score is **deuce**. When at least three points have been scored by each side and a player has one more point than his opponent, the score of the game is **advantage** for the player in the lead.*

(a) Find, as a function of p, the probability that player1 wins a game over player2 without any deuces.

(b) Find, as a function of p, the probability that player1 wins a game over player2 with only one deuce.

(c) *Bonus question.* Find, as a function of p, the probability that player1 wins a game over player2.

MARKS : 1. [25] **2.** [30] **3.** [25] **4.** [20] Bonus question [10]

Lebanese American University **Spring 2008**

Byblos

Probability and Statistics **Date:** 29/03/2008

Test #1 **Duration:** 1h 30

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1. A man comes home. He has 8 keys on his keychain, but he is drunk he can't remember which is the key to the front door. So he randomly chooses one key after the other until he picks the right one. Find the probability that the man picks the right key on the fifth try if

   (i) the man is so drunk that he is likely to pick the same key again even though he just tried it.

   (ii) the man is not so drunk so he tries a key only once.

   Conclusion?

2. In an NBA (National Basketball Association) championship series, the team which wins four games out of seven will be the winner. Assume that Chicago Bulls has probability p of winning over Indiana Pacers, and that both teams face each other in the championship games.

   (a) Find the probability that Chicago Bulls will win the series in five games?

   (b) Find the probability that Chicago Bulls will lose the series in six games?

   (c) Find the probability that Chicago Bulls will win the series?

3. We consider two dice: one red and one blue. We assume that

   - the red die is loaded in such a way that an even number is thrice as likely to occur as an odd number,
   - the blue die is loaded in such a way that an odd number is thrice as likely to occur as an even number.

   We toss the two dice five times.

   (a) Find the probability that we obtain the same even sum in the five tosses.

   (b) Find the probability that we obtain an even sum in the five tosses.

   (c) Find the probability that we obtain exactly three even sum in the five tosses.

   (d) Find the probability that we obtain an odd number of even sum in the five tosses.

4. We consider a bag which contains 5 red balls, 3 blues balls and 7 yellow balls. We also consider a loaded die in such a way that an even number is twice as likely to occur as an odd number. We consider the following two consecutive experiments: ⟶

- We draw in succession and with replacement four balls from the bag.
- We toss the die several times to obtain $n+1$ times an even number for the first time, where n is the number of obtained blue balls in the first experiment.

(a) Find the probability that we obtain exactly two blue balls in the first experiment.

(b) Find the probability that we obtain exactly two blue balls in the first experiment and that we toss the die four times in the second experiment.

(c) Find the probability that we toss the die four times in the second experiment.

5. *Bonus question.* We consider four dice, one red, one blue, one green and one yellow. We assume that:

   - the red die is loaded in such a way that an even number is twice as likely to occur as an odd number,
   - the blue die is loaded in such a way that an even number is thrice as likely to occur as an odd number.
   - the green and the yellow dice are fair.

   We toss the four dice. Find the probability that we obtain a same number a on the red and the green dice and a same number b on the blue and the yellow dice where $a \neq b$.

MARKS : 1. [15] **2.** [25] **3.** [30] **4.** [30] **5.** [10]

Lebanese American University	**Summer II**
Byblos	**2008**

Probability and Statistics **Date:** 25/08/2008
Test #1 **Duration:** 1h 30

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1. A mathematics professor wishes to schedule an appointment with each of his 10 teaching assistants, 5 men and 5 women, to discuss his calculus course.
   (a) What is the probability that at least one male assistant is among the first 4 with whom the professor meets.
   (b) What is the probability that after the first 7 appointments, he has met with all female assistants.

2. Assume that you have 8 (different) songs in your mp3 player: 3 for Michael Jackson, 2 for Bryan Adams and 3 for Phil Collins. We also assume that you use the shuffle function, that is, the mp3 player plays the songs in random order and it plays each song once before it starts repeating.
   A. In this part we assume that you only have time to listen to 6 songs.
      i. Find the probability the the first and the last songs are for Michael Jackson.
      ii. Find the probability the the first and the last songs are for the same artists.
      iii. *Bonus question.* Find the probability that the listened songs of each artists follow each other.
   B. In this part we assume that you only have time to listen to 8 songs.
      i. Find the probability that the first and the last song are for Michael Jackson.
      ii. Find the probability the the first and the last songs are for the same artists.
      iii. Find the probability that the listened songs of each artists follow each other.
   C. In this part we assume that you only have time to listen to 12 songs.
      i. Find the probability that the first and the last song are for Michael Jackson.
      ii. Find the probability the the first and the last songs are for the same artists.
      iii. *Bonus question.* Find the probability that the listened songs of each artists follow each other.

3. We consider the following three kinds of coins:
   - *First kind*: Coins loaded in such a way that $P(H) = 2P(T)$.
   - *Second kind*: Coins loaded in such a way that $P(T) = 2P(H)$.
   - *Third kind*: Coins with heads in both sides.

   Now we consider a box which contains two coins of the first kind, three coins of the second kind and two coins of the third kind. We select a coin from the box and we flipped. After we select another coin from the box and we flipped.
   (a) Find the probability that we obtain two heads. $\longrightarrow$

(b) Given that we obtain two heads, find the probability that the two selected coins were of the second and the third kind.

MARKS : 1. [30] **2.** [40] **3.** [30] Bonus questions [10]

Lebanese American University **Spring 2009**

Byblos

Probability and Statistics **Date:** 06/04/2009
Test #1 **Duration:** 2h

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1. The probability that a person selected randomly from a population will exhibit the classic symptom of a certain disease is 0.2 and the probability that a person selected at random has the disease is 0.23. The probability that a person who has the symptom also has the disease is 0.18. A person selected at random from the population does not have the symptom: What is the probability that the person has the disease?

2. A university has 90 faculty members: 60 male and 30 female. How many committees of size 18 can we form if:
   (a) no restriction?
   (b) among the selected 18 faculty members, one position is president and one is vice president?
   (c) among the selected 18 faculty members, one position is president and tow are vice presidents?
   (d) among the selected 18 faculty members, one position is president, one is a male vice president and one is a female vice president?

3. Assume that a coach is training 18 students: 12 boys and 6 girls. He wants to form 6 lines of 3 students each (left-wing, center, and right-wing). Assume that the order of assigning these positions matters.
   (a) What is the probability that the first four lines contain only boys?
   (b) What is the probability that the first and the last line contain only boys?
   (c) What is the probability that each line contains students of the same sex?
   (d) What is the probability that each girl is between two boys?
   (e) What is the probability that both Alain and Mike are in the same line?
   (f) *Bonus question.* What is the probability that there is only one line which contains at least two girls?

4. We consider a bag which contains 5 red balls and 7 blue balls. We draw at the same time two balls from the bag. If the drawn balls have the same color then we put a new red ball in the bag, and if the drawn balls have different color then we put a new blue ball in the bag. Finally we draw one ball from the bag.
   (a) Find the probability that the last drawn ball is red?
   (b) Given that the last drawn ball is red, find the probability that the first two drawn balls have the same color?

MARKS : 1. [15] **2.** [25] **3.** [35] **4.** [25] **Bonus question.** [10]

Lebanese American University	Summer II
Byblos	2009

Probability and Statistics **Date:** 31/08/2009
Test #1 **Duration:** 1h 30

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1. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Find the probability that a person has the disease given that the test indicates the presence of the disease.

2. The probability that a randomly chosen male has a circulation problem is 0.25. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the probability that a male has a circulation problem, given that he is a smoker?

3. Assume that we have 20 different books such that 5 books are for the same author A. In how many ways can we put these 20 books on a rectilinear bookstore if:
   (a) No restriction.
   (b) The five books of the author A follow each other.
   (c) Only four books of the author A follow each other (that is, four books of the author A follow each other but the five books of the author A do not follow each other).
   (d) At least two books of the author A follow each other.

4. Assume that a coach is training 18 students: 12 boys and 6 girls. He wants to form 6 rows of 3 students each (left-wing, center, and right-wing). Assume that the order of assigning these positions matters.
   (a) What is the probability that the first four rows contain only boys?
   (b) What is the probability that the first and the last row contain only boys?
   (c) What is the probability that each row contains students of the same sex?
   (d) What is the probability that each girl is between two boys?
   (e) What is the probability that both Alain and Mike are in the same row?
   (f) *Bonus question.* What is the probability that there is only one row which contains at least two girls?

MARKS : 1. [15] **2.** [20] **3.** [30] **4.** [35] Bonus questions [10]

Lebanese American University **Spring 2010**

Byblos

Probability and Statistics **Date:** 24/03/2010

Test #1 **Duration:** 2h

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1. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.

2. A student must choose exactly two out of three electives: art, French, and mathematics. He chooses art with probability 5/8, French with probability 5/8, and art and French together with probability 1/4.

   (a) What is the probability that he chooses mathematics?

   (b) What is the probability that he chooses either art or French?

3. A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face. (For example, a six is three times as probable as a two.) We toss the die three times, find the probability that we obtain two even numbers and one odd number?

4. We consider seven distinct boxes such that each box can contain at most seven items. Seven distincts items will be distributed in these boxes.

   A. In this part we assume that there is no order inside a box.

      i. Find the probability that the seven items are placed in the same box.

      ii. Find the probability that the seven items are placed in seven boxes.

      iii. Find the probability that the seven items are placed in six boxes.

      iv. Find the probability that two specifc items are placed in the same box.

   B. In this part we assume that there is an order inside each box.

      i. Find the probability that the seven items are placed in the same box.

      ii. Find the probability that the seven items are placed in seven boxes.

      iii. Find the probability that the seven items are placed in six boxes.

      iv. Find the probability that two specifc items are placed in the same box.

5. *Bonus question.* Karim and Mira are taking a mathematics course. The course has only three grades: A, B, and C. The probability that Karim gets a B is 0.3. The probability that Mira gets a B is 0.4. The probability that neither gets an A but at least one gets a B is 0.1. What is the probability that at least one gets a B but neither gets a C?

MARKS : 1. [20] **2.** [20] **3.** [20] **4.** [40] **5.** [10]

Lebanese American University	**Summer II**
Byblos	**2010**

Probability and Statistics	**Date:** 12/08/2010
Test #1	**Duration:** 1h 45

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1. Janice, Tom, John, and Tamira are trying to decide on who will make dinner and who will wash the dishes afterwards. They randomly pull two names out of a hat to decide, where the first name drawn will make dinner and the second will do the dishes. Determine the probability that the two people pulled will have first names beginning with the same letter. Assume the same person cannot be picked for both.

2. A public health researcher examines the medical records of a group of 937 men who died in 2005 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease. Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

3. A study is being conducted in which the health of two independent groups of 5 policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 4 participants complete the study in one of the two groups, but not in both groups?

4. (a) Two tour guides are leading 12 tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. How many different grouping of guides and tourists are possible?
   (b) Repeat (a) but in the case in which three tour guides are leading the 12 tourists.
   (c) *Bonus question.* Repeat (a) but in the case in which four tour guides are leading the 12 tourists.

5. Let $N = 3600000$.
   (a) How many divisors does N have?
   (b) How many even divisors does N have?
   (c) How many divisors of N are multiples of 6?
   (d) How many divisors of N are multiples of 30?

   (*Hint*: Remark that $N = 2^7\,3^2\,5^5$)

MARKS : 1. [20] **2.** [20] **3.** [20] **4.** [20] **5.** [20] **Bonus question.** [10]

Lebanese American University **Spring 2011**

Byblos

Probability and Statistics **Date:** 24/03/2011

Test #1 **Duration:** 2h

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1. There is a new diagnostic test for testing quality of computer chip. It is designed to be used for a manufacture which has about 0.05% of defective chips. The test is not perfect but will detect a bad computer chip 95% of the time. It will, however, say that a good computer chip is bad about 4% of the time. If a computer chip produced by this manufacture is selected at random and the test indicates that it is bad, what is the probability that this selected computer chip is really a bad chip?

2. A real estate agent has 11 keys to open 10 new homes. Each key can open exactly one home except one master key which can open all the new homes. If 40% of these new homes are usually left unlocked, what is the probability that the real estate agent can get into a specific home if the agent selects 3 keys at random before leaving the office?

3. From the set $\{1, 2, ..., 59, 60\}$ we select in succession and with replacement two numbers.
   (a) Find the probability that the two selected numbers are the same?
   (b) Find the probability that the first selected number is less than the second selected number?
   (c) What will be the answer of the preceding question if the selection was in succession but without replacement? Explain.

4. A license plate in a certain state consists of 4 digits, not necessarily distinct, and 3 letters, also not necessarily distinct.
   (a) How many distinct license plates are possible if no restriction?
   (b) How many distinct license plates are possible if it must begin and terminate by a digit?
   (c) How many distinct license plates are possible if it must begin and terminate by a letter?
   (d) How many distinct license plates are possible if the three letters must appear next to each other?
   (e) How many distinct palindrome license plates are possible?
   (A palindrome license plate is a license plate that reads the same from left to right as right to left)

5. Let A and B be two events such that:
$$P(A) = 1/2 \text{ and } P(A|B) = P(A'|B') = 2/3$$
   Find $P(B)$.

MARKS : 1. [20] **2.** [25] **3.** [25] **4.** [30] **5.** [10]

Lebanese American University **Spring 2012**

Byblos

Probability and Statistics **Date:** 23/03/2012
Test #1 **Duration:** 2h

Name: **ID:**

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1. In a study of pleas and prison sentences, it is found that 45% of the subjects studied were sent to prison. Among those sent to prison, 40% chose to plead guilty. Among those not sent to prison, 55% chose to plead guilty. If a study subject is randomly selected and it is then found that the subject entered a guilty plea, find the probability that this person was not sent to prison.

2. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.

3. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:
   - 14% have high blood pressure.
   - 22% have low blood pressure.
   - 15% have an irregular heartbeat.
   - Of those with an irregular heartbeat, one-third have high blood pressure.
   - Of those with normal blood pressure, one-eighth have an irregular heartbeat.

   (a) What is the probability that a selected patient who has irregular heartbeat also has low blood pressure?

   (b) What is the probability that a selected patient has a regular heartbeat and low blood pressure?

4. A garage door opener has a ten-digit keypad. Codes to the door must consist of 5 digits with no adjacent digits the same.

   (a) How many codes are possible if no restriction?

   (b) How many palindrome codes are possible? (A palindrome code is a code which reads the same from left to right as right to left)

   (c) How many codes are possible if the first and the third digits are the same?

   (d) How many codes are possible if the first and the last digits are the same?

5. You and 9 of your friends are at a restaurant where they serve 12 different meals. Suppose that you and your friends all order a meal and that the different meals are all equally likely to be chosen.

   (a) What is the probability that no one orders the first meal on the list?

   (b) What is the probability that at least two persons order the same meal?

   (c) What is the probability that only two out of your 9 friends order the same meal as you?

   (d) What is the probability that the 10 persons (you and your 9 friends) order exactly two meals from the list?

MARKS : 1. [20] **2.** [20] **3.** [20] **4.** [25] **5.** [25]

Lebanese American University **Fall 2012**

Byblos

Probability and Statistics **Date:** 29/10/2012
Test #1 **Duration:** 1h 45

Name: **ID:**

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1. Prove that if A and B are independent events, then A' and B' are also independent.

2. There is a new drug test for heroin. It is designed for a population in which 3% are heroin users. The test correctly identifies users 95% of the time and correctly identifies nonusers 90% of the time. If a randomly selected person has a positive test result, what is the probability that he is a heroin user?

3. Suppose there are two restaurants in a small town, restaurant A and restaurant B. Suppose further that 20% of the people in the town dine at neither restaurant. Of those that go to at least one of the restaurants, 75% dine in both. Of those that dine at exactly one of the restaurants, 75% dine at restaurant A.

   (a) What proportion dine at restaurant B?

   (b) Of those that dine at restaurant B, what proportion dine at restaurant A?

4. A class room contains 25 persons, 15 women and 10 men. We select 10 persons at random. In addition, assume that the room contains exactly two married couples. Find the probability that:

   (a) The 10 persons consist of exactly 5 women and 5 men?

   (b) At least one man is selected?

   (c) The 10 persons contain at least one married couple?

5. In 1693, Samuel Pepys wrote Isaac Newton to ask which of the following three events is more likely:

   (A) Getting at least one 6 when six dice are rolled.

   (B) Getting at least two 6 when twelve dice are rolled.

   (C) Getting at least three 6 when eighteen dice are rolled.

   What is the answer? (Pepys initially thought that (C) had the highest probability).

6. Ten pair of shoes are kept in a rack and one of these pairs is yours. If four shoes are selected at random, what is the probability that:

   (a) Your pair is among them?

   (b) There are two pairs among them?

   (c) There is at least one pair among them?

MARKS : 1. [8] **2.** [17] **3.** [20] **4.** [22] **5.** [21] **6.** [22]

Lebanese American University **Spring 2015**

Byblos

Probability and Statistics **Date:** 24/02/2015
Test #1 **Duration:** 1h 30

Name: **ID:**

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1. Assume that:

   - The probability that a randomly selected person in a typical local bar environment is born within 25 miles of Chelsea is 0.05.
   - The probability that a person born within 25 miles of Chelsea actually supports Chelsea F. C. is 0.7.
   - The probability that a person is not born within 25 miles of Chelsea and does not support Chelsea F. C. is 0.8.

   While watching a game of Champions League football in a cafe, you observe someone who is clearly supporting Chelsea F. C. in the game. What is the probability that they were actually born within 25 miles of Chelsea?

2. There is a new drug test for heroin. It is designed for a population in which 3% are heroin users. The test correctly identifies users 95% of the time and correctly identifies nonusers 90% of the time. If a randomly selected person has a positive test result, what is the probability that he is a heroin user?

3. In 1693, Samuel Pepys wrote Isaac Newton to ask which of the following three events is more likely:

   (A) Getting at least one 6 when a die is rolled six times.

   (B) Getting at least two 6 when a die is rolled twelve times.

   (C) Getting at least three 6 when a die is rolled eighteen times.

   What is the answer? (Pepys initially thought that (C) had the highest probability).

4. Eighteen chairs in a row are to be occupied by twelve students and six professors. Find the probability that:

   (a) The six professors follow each other.

   (b) The six professors follow each other or the twelve students follow each other.

   (c) Each professor is between two students.

5. A class room contains 25 persons, 15 women and 10 men. We assume that among these 25 persons, there are exactly two married couples. We select, with no replacement, 10 persons at random. Find the probability that:

   (a) The 10 persons consist of exactly 5 women and 5 men?

   (b) At least one man is selected?

   (c) The 10 persons contain at least one married couple?

6. Let A and B be two events. We denote by C the symmetric difference between A and B, that is, $C = (A \cap B') \cup (A' \cap B)$. We assume that $P(A' \cap B') = 0.2$, $P(A \cap B | A \cup B) = 0.75$ and $P(A|C) = 0.75$. Find $P(B)$.

MARKS : 1. [15] **2.** [15] **3.** [20] **4.** [24] **5.** [24] **6.** [12]