Lebanese American University **Fall 2010**

Byblos

Discrete Structure I **Date:** 27/01/2011
Final Exam **Duration:** 2h

Name: **ID:**

Part I: Problems (70 pts)

1. **[10 pts]** Find a resolution proof and a formal proof for the following propositional logic formula:

$$\{A \longrightarrow B\,;\, \neg A \longrightarrow C\,;\, C \longrightarrow D\} \vdash \neg B \longrightarrow D$$

2. **[15 pts]** Prove, by induction, the following:

   (a) $3^{6n} - 2^{6n}$ is always divisible by 35 for all $n \in \mathbb{N}^*$.

   (b) $\displaystyle\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n}$.

3. **[10 pts]** Solve using pigeonhole principle the following two independent questions:

   (a) A computer network consists of eight computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same numbers of other computers.

   (b) 1335 people are seated in a row of 2001 chairs. Prove that there are 3 consecutive non-empty chairs.

4. **[10 pts]** We consider the following program (input is n and output is r):

```
r:=0;
if n > 0 then
   begin r := 1;
         i := 1;
         while i < n do
           begin i := i + 1;
                 r := r + 3*i*i - 3*i + 1;
           end;
   end;
```

(a) Find the output of the program for $n = 1$, $n = 2$ and $n = 3$.

(b) Which function computes the program?

5. **[15 pts]** Verify, in the following cases, if the binary relation R defined over the set X is an equivalence relation or not. If R is an equivalence relation then find the set of classes π_R.

   (a) $X = \mathbb{R}$ and $x\,R\,y$ iff $|x| + |y| = |x + y|$.

   (b) $X = \mathbb{R}^*$ and $x\,R\,y$ iff $xy > 0$.

   (c) $X = \mathbb{R}$ and $x\,R\,y$ iff $x^2 - xy + 2x - 2y = 0$.

6. **[10 pts]** Find for the following two ordered sets the maximal, minimal, greatest and least elements (if any):

   (a) $X = \{2, 3, 6, 12, 18, 24\}$ and the order is $x\,R\,y$ iff x divides y.

   (b) $X = \{\{2\}, \{2, 3\}, \{2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}$ and the order is $x\,R\,y$ iff $x \subseteq y$.