Lebanese American University **Summer I**

Byblos **2007**

Calculus IV **Date:** 30/07/2007
Final exam **Duration:** 2h

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1. We consider the velocity field $\mathbf{F} = (ze^x + e^y)\,\mathbf{i} + (xe^y + e^z)\,\mathbf{j} + (ye^z + e^x)\,\mathbf{k}$.
   (a) Prove the $\mathbf{F}$ is conservative.
   (b) Find a potential function for the field $\mathbf{F}$.
   (c) Find the flow of $\mathbf{F}$ over the curve $\mathbf{r}(t) = \ln(1+t^2)\,\mathbf{i}+\ln(2+t)\,\mathbf{j}+\ln(3+2t^2)\,\mathbf{k}$ from $t = 0$ to $t = 1$.

2. We denote by V the volume of the region bounded below by the the paraboloid $z = x^2 + y^2$ and above by the sphere $x^2 + y^2 + z^2 = 2$.
   (a) Express, without evaluating, the volume V as a triple integral using:
      i. the rectangular coordinates,
      ii. the cylindrical coordinates,
      iii. the spherical coordinates.
   (b) Find the volume V by evaluating one of the three triple integrals.

3. Find the extreme values of the function $f(x, y, z) = x - y + z$ on the sphere unit sphere $x^2 + y^2 + z^2 = 1$.

4. We consider the function $f(x, y) = x^4 + y^4 - 4(x - y)^2$.
   (a) Test the function $f(x, y)$ for local maxima and minima and saddle points.
   (b) Prove that the point where the test fails in (a) is in fact a saddle point. (*Hint*: Find $f(x, x)$ and $f(x, 0)$ and conclude.....)

5. Find the integral of $f(x, y, z) = x^2 - y + z$ over the curve $C = C_1 \cup C_2$ where
   - C_1 is the part of the curve $x^2 + y^2 = 1$ in the $y \geq 0$ part of the xy-plane from $(-1, 0, 0)$ to $(1, 0, 0)$
   - C_2 is the line segment in xz-plane from $(1, 0, 0)$ to $(0, 0, 1)$

6. Find the area of the band cut from the paraboloid $x^2 + y^2 - z = 0$ by the planes $z = 2$ and $z = 6$.

7. *Bonus question.* Find the largest volume of a rectangular box if its surface is known to be 24.

MARKS : 1. [15] **2.** [30] **3.** [15] **4.** [15] **5.** [10] **6.** [15] **7.** [10]